

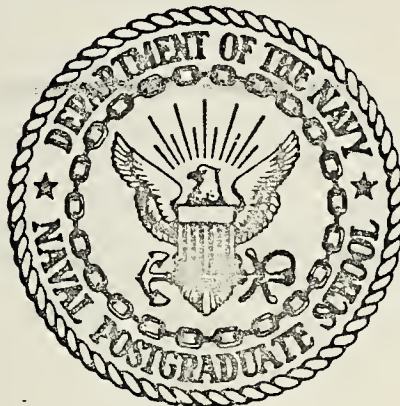
**EFFECTS OF ATMOSPHERIC TURBULENCE ON LASER  
COMMUNICATIONS**

**Thomas Prater Rankin**



# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



# THESIS

EFFECTS OF ATMOSPHERIC TURBULENCE  
ON LASER COMMUNICATIONS

by

Thomas Prater Rankin .

June 1974

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Effects of Atmospheric Turbulence  
on Laser Communications

by

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Submitted in partial fulfillment of the  
requirements for the degree of

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## ABSTRACT

A multiplicative model based on the amplitude modulation equation is developed to predict the phenomenon of frequency translation effects of atmospheric scintillation on AM laser communications. An experiment to measure the effects of scintillation on atmospheric laser communications is designed and built. Preliminary results of the experiment are in general agreement with the predictive model. The best agreement is in predicting the scintillation components present in the AM frequency spectrum with some questions raised about the ability of the model to predict the bandwidth of the translated scintillation noise.



## TABLE OF CONTENTS

I.	INTRODUCTION .....	6
II.	PROJECT BACKGROUND .....	8
III.	THEORY .....	9
	A. GENERAL REMARKS .....	9
	B. TURBULENCE THEORY .....	11
	C. MODULATION THEORY .....	20
IV.	EXPERIMENTAL APPARATUS .....	24
	A. THE EXPERIMENTAL SETUP .....	24
	B. THE LASER AND MODULATOR SYSTEM .....	26
	C. THE OPTICAL RECEIVER SYSTEM .....	27
	D. SIGNAL PROCESSING EQUIPMENT .....	27
V.	DATA ANALYSIS .....	28
VI.	CONCLUSIONS .....	31
APPENDIX A	FIGURES .....	33
BIBLIOGRAPHY	.....	46
INITIAL DISTRIBUTION LIST	.....	48



## I. INTRODUCTION

Much of the current research in laser communications systems is directed towards atmospheric transmission of intelligence at optical frequencies. There are many advantages for both military and civilian applications of laser communications systems. Some of the more obvious advantages of an atmospheric optical communications system are security (because of the systems inherent directionality) and bandwidth (data rates of  $10^{14}$  bits/sec are within the state-of-the-art). As an example of a special type of problem that is readily solved by atmospheric optical communications systems, consider the so-called blackout that occurs in rocket or nuclear generated plasmas below the plasma frequency of approximately  $10^{11}$  Hz. Optical systems in the frequency range  $10^{13}$ - $10^{15}$  Hz readily penetrate the plasma blackout. [Ref. 1]

While the advantages of using atmospheric optical communications readily justify the intense interest in the subject, there are several rather severe problems associated with using the atmosphere as an optical transmission medium. Many techniques have been proposed and investigated which offer promise in overcoming many of the problems associated with atmospheric transmission, but at this writing, atmospheric effects are still capable of "shutting down" most potential atmospheric optical links.

This work was the first in an ongoing series of experiments in which the objective is to model atmospheric effects on a laser communications system and to experimentally determine these effects. Atmospheric effects on laser beam modulation and propagation fit into three general categories: refraction, absorption and particle scattering. In general the emphasis of this first series of experiments is on refraction, and in particular





on small scale refraction, called scintillation, in which the laser beam is differentially focused and defocused causing localized intensity differentials in the beam. The scope of this particular project was to design an experiment to propagate a laser beam amplitude modulated at various frequencies in the turbulent atmosphere and to recover the intelligence on the beam in the presence of severe scintillation effects. Signal processing techniques were then used on the recovered signal to study the effects of scintillation and its relation to the intelligence.



## II. PROJECT BACKGROUND

This series of experiments, directed by Dr. John Powers, is taking place in conjunction with an atmospheric laser propagation project funded by the Naval Ordnance Laboratory. The NOL project is divided into two related areas, meteorology and optics. The meteorological research group, headed by Dr. Kenneth Davidson of the Naval Postgraduate School, makes and evaluates the micrometeorological measurements of atmospheric turbulence. The optical research group, headed by Dr. Eugene Crittendon, makes and evaluates the optical measurements of laser beam scintillation.

Scintillation, random intensity fluctuations in a light beam, is a particularly critical characteristic in an atmospheric laser communications system. The end result of most atmospheric AM laser communications systems is intensity modulation. Also FM systems are operated above some intensity threshold. Since the effect of scintillation is random intensity variations on the beam, the result is random interference with the received signal whenever the signal strength falls below this threshold.



### III. THEORY

#### A. GENERAL REMARKS

A description of scintillation and its effects on optical modulation and propagation in the atmosphere must necessarily be a description of atmospheric turbulence.

Consider the spectrum of horizontal wind velocity published by Van der Hoven in 1957. [Ref. 16] As Figure (1) shows the atmosphere is never still. The consequence is that there are constant variations in temperature and pressure which affect the refractive index of the atmosphere. Large scale variations are responsible for beam wander in space and small scale variations are responsible for scintillation. Haagensen [Ref. 2] found that the scintillation frequency spectrum extended out to 300 Hz and had a dynamic range of 40 db. There have been indications on the current project [Ref. 3] that the scintillation spectrum can extend as far as 2Khz.

While turbulence is very difficult to define and agreement is not universal, Lumley and Panofsky [Ref. 4] list several properties of turbulence. The most important properties of turbulence are that it is nonlinear, diffusive, three dimensional and random. The complexity of the turbulent regime precludes the differential equation approach to effectively describing it; instead the random variable approach is used. The effects of turbulence on optical propagation and modulation are described in terms of the random fluctuation in the refractive index.

The result of interest will be a model of an atmospheric laser communication system which predicts the effect of small scale turbulence on the information content of a modulated laser beam. In order to make



such a prediction, there must first be a description of the effect of turbulence on the propagating laser beam. [Ref. 2] Then the applicable modulation expression must be combined with the propagation equation to arrive at an expression which predicts the effects of turbulence on the modulated signal.

The problem of describing the relation between propagation and turbulence has three parts. There must be a workable characterization of atmospheric turbulence; also it is necessary to describe the propagation of energy in a random medium in terms of Maxwell's equations, and finally the first two results must be combined to arrive at an analytic relationship between the statistically random refractive index and the phase and amplitude of the propagating beam. Substituting an expression which includes a modulated refractive index as well as a random component will complete the development.

The complete, step-by-step derivation is very long and at times obtuse, therefore only intermediate and final results of interest will be covered here. A more complete analysis of the relationship between propagation and turbulence is in Reference 2 and a complete treatment may be found in References 4,5,7,8 and 9.

In view of the, at times, rather complicated mathematics which are to follow, it seems pertinent at this time to first attempt a qualitative description of the process to be covered by the mathematics. The theory underlying random turbulent motion of the atmosphere says that turbulent motion is the vehicle for transferring energy from large scale phenomena such as wind shear and near ground convection through smaller and smaller scale phenomena until the energy is dissipated in viscous friction. One of the basic assumptions of the theory is that the mechanism for transferring





energy is a closed process. Professor Kenneth Davidson of the Naval Postgraduate School has rather aptly described the phenomenon in the following manner:

"Big whirls have little whirls that feed upon their velocity.  
Little whirls have lesser whirls and so on to viscosity".

If the process is a closed one, then energy must enter at some level of the "big whirls" characterized by some maximum scale size, say  $L_0$ . Just as the upper level of the process is described by a scale size, so are the lowest levels of turbulence or least "whirls" just above viscous dissipation characterized by a scale size designated by  $l_0$ . Typical values of  $L_0$  are a few meters to a few tens of meters; typical values of  $l_0$  are a few millimeters. The turbulence regime described above characterized by a scale with a size range  $l_0 < l < L_0$  is called the inertial subrange. It is the minute pressure and temperature differentials in the inertial subrange that cause random fluctuations in the index of refraction of the atmosphere known as scintillation.

## B. TURBULENCE THEORY

Consider a continuous random medium whose index of refraction,  $n(r,t)$ , is a continuous random point function which characterizes the transmission medium. Transit times of line-of-sight optical transmission paths are relatively short compared to the time constant of the spatial distribution of the refractive index, about  $10^{-3}$  sec. Therefore with minimal error the time dependence of  $n(r,t)$  can be expressed by  $n(r,t_i)$ ,  $i = 1,2,3,\dots$ , each being a single event of the random process. Since the mean value of the refractive index of the atmosphere is very near 1,



$$\bar{n}(\bar{r}) = \langle n(\bar{r}) \rangle + n_1(\bar{r}) \approx 1 + n_1(\bar{r}), \quad (1)$$

where the brackets  $\langle \rangle$ , denote the mean value, and  $n_1(\bar{r})$  is the random fluctuation from the mean.

The power spectrum of the turbulence is a function of the size of the turbulence. The relationship can be described in terms of a spatial wave number  $k$  given by

$$k = \frac{2\pi}{l}, \quad (2)$$

where  $l$  is the size of the turbulent eddy in the inertial subrange,  $l_0 < l < L_0$ . Tatarski [Ref. 6] assumed that the power spectrum of the turbulence within this subrange was given by the empirical Kolomogorov model,

$$\phi_n(k) = 0.033 C_N^2 k^{-\frac{11}{3}} \exp(k^2/k_m^2). \quad (3)$$

$k_m$  is the spatial wave number associated with the inner scale (i.e.,  $k_m = 2\pi/l_0$ ) and  $C_N^2$  is a constant that parameterizes the total energy in the turbulence. Because of the importance of  $C_N^2$  in the description of the turbulent regime, the main thrust of the turbulence theory portion of this derivation is towards an expression for  $C_N^2$  in terms of quantities that can easily be measured experimentally.

By considering the refractive indicies at two points on a line normal to the mean wind direction separated by a distance  $d$  which is some value less than  $L_0$ , Tatarski empirically developed an expression for a statistical estimator,  $D_n(d)$  which he also related to the refractive index.

$$D_n(d) = C_N^2 d^{\frac{2}{3}} \quad (4)$$



$$D_n(d) = \langle [n(\bar{r}) - n(\bar{r} + d)]^2 \rangle \quad (5)$$

Eliminating  $D_n(d)$  from Equations (4) and (5) yields an expression for  $C_N$  in terms of the refractive index and some experimental lengths,  $d$ , bounded in the inertial subrange.

$$C_N^2 = \frac{\langle [n(\bar{r}) - n(\bar{r} + d)]^2 \rangle}{d^{\frac{2}{3}}} \quad (6)$$

From the manner in which they were defined above, the functional arguments  $r$  and  $d$  are physically vectors but by assuming that, in the inertial subrange, turbulence is isotropic and homogeneous, they can be considered as scalars.

For homogeneous, stationary conditions, the covariance of the refractive index is related to the turbulent energy spectrum by the three dimensional Fourier transform pair:

$$C_n(d) = \langle n(r+d) n(r) \rangle \quad (7a)$$

$$C_n(d) = \frac{4\pi}{d} \int_0^\infty k \phi_n(k) \sin kd \, dk. \quad (7b)$$

Tatarski also showed that the structure function,  $D_n(d)$  given by Equations (4) and (5), is related to the energy spectrum by the Fourier transform

$$D_n(d) = 8\pi \int_0^\infty \left(1 - \frac{\sin kd}{kd}\right) \phi_n(k) k^2 dk, \quad (8)$$

given conditions of isotropy, homogeneity and stationarity. From Equations (3) and (7b) one can relate  $C_n(d)$ , and  $C_N$  which can then be used to



describe the fluctuations of the refractive index of the atmosphere. Similar but less complicated approaches to the problem are covered in Refs. 14 and 15.

Digressing for the moment from atmospheric statistics, the equations that govern the propagation of electromagnetic energy through a random medium will be developed. Consider Maxwell's scalar wave equation written in terms of the refractive index,  $n(r)$ . Under the assumption of Equation (1), i.e., small perturbations in the refraction index, Maxwell's equation can be written as

$$\nabla^2 E + k^2 n^2 E = 0 \quad (9)$$

where  $E$  is any component of the complex valued electric field, and where the time dependence,  $e^{j\omega t}$ , is understood.

Because of the wide dynamic range of intensity fluctuations in the scintillation, it is conventional at this time to express Equation (9) in terms of the logarithm of the electric field. Dividing Equation (9) by  $E$  yields

$$\frac{\nabla^2 E}{E} + k^2 n^2 = 0. \quad (10)$$

Noting that

$$|\nabla \ln E| = \frac{1}{E} \nabla E, \quad (11)$$

by squaring Equation (11) we arrive at

$$|\nabla \ln E|^2 = \frac{1}{E^2} (\nabla E)^2 \quad (12)$$





Taking another derivitive of Equation (11) yeilds

$$\nabla^2 \ln E = \frac{-1}{E^2} (\nabla E \cdot \nabla E) + \frac{1}{E} \nabla^2 E , \quad (13)$$

which can be rewritten as

$$\nabla^2 \ln E = - \frac{1}{E} (\nabla E)^2 + \frac{1}{E} \nabla^2 E . \quad (14)$$

Substituting Equation (12) into Equation (14) yields

$$\nabla^2 \ln E = - |\nabla \ln E|^2 + \frac{1}{E} \nabla^2 E . \quad (15)$$

Since from Equation (10)

$$\frac{1}{E} \nabla^2 E = k_n^2 , \quad (16)$$

Equation (15) can be rewritten as

$$\nabla^2 \ln E + |\nabla \ln E|^2 + k_n^2 = 0 \quad (17)$$

which is the logarithmic form of the scalar wave equation.

At this point Tatarski [Ref. 6] defined the electric field in terms of its amplitude,  $A(r)$  and phase  $S(r)$  to be

$$E(r) = A(r) e^{jS(r)} . \quad (18)$$

He then defined the complex phase  $\Psi(r)$  by

$$E(r) = e^{\Psi(r)} = e^{\ln A(r) + jS(r)} , \quad (19)$$



where  $\text{Re}\Psi = \ln A(r)$  and  $\text{Im}\Psi = S(r)$ . Equations (17) and (19) will allow the field to be described in terms of the logarithm of its amplitude which, because of the wide dynamic range of the amplitude, is a much more easily measured quantity.

By substituting Equation (19) into Equation (17) the wave equation can be written in terms of the complex phase and the random refractive index.

$$\nabla^2 \Psi + |\nabla \Psi|^2 + k^2 [1 + n_1(r)] = 0 \quad (20)$$

If the complex phase,  $\Psi$ , is separated into two components, the mean value  $\Psi_0$ , and the random fluctuation about the mean,  $\Psi_1$ , then

$$\Psi = \Psi_0 + \Psi_1 \quad (21)$$

The electric field can then be represented by

$$E = e^{\Psi_0 + \Psi_1} = E_0 e^{\Psi_1} \quad (22)$$

where  $E_0$  is the mean of the electric field and satisfies

$$\nabla^2 E_0 + k^2 E_0 = 0 \quad (23)$$

or the transformed equation

$$\nabla^2 \Psi_0 + |\nabla \Psi_0|^2 + k^2 = 0. \quad (24)$$

By substituting Equation (19) into Equation (24) we arrive at

$$\nabla^2 \Psi_0 + |\nabla \Psi_0|^2 + k^2 = 0. \quad (25)$$



Substituting Equation (21) into Equation (20), subtracting the result from Equation (25) yields

$$\nabla^2 \Psi_1 + |\nabla \Psi_1|^2 + 2\nabla \Psi_0 \cdot \nabla \Psi_1 + k^2(2n_1 + n_1^2) = 0 \quad (26)$$

This is the nonlinear Ricatti equation. Since  $n_1$  is much less than 1,  $k^2 n_1^2$  is much less than  $2k^2 n_1$  and may be neglected. Similarly, since  $|\nabla \Psi_1| \ll |\nabla \Psi_0|$  which is of the order of  $k$ , the  $|\nabla \Psi_1|^2$  term can also be neglected. The final form of Equation (26) then is

$$\nabla^2 \Psi_1 + 2\nabla \Psi_0 \cdot \nabla \Psi_1 + 2k^2 n_1^2 = 0 \quad (27)$$

The assumption that  $|\nabla \Psi_1| \ll |\nabla \Psi_0| \sim 2k$  is the only significant assumption (originally made by Rytov). It is equivalent to saying that the change in the variation of the complex phase  $\Psi$ , over a distance of about one wavelength is much less than  $2\pi$  radians. Realizing that the optical wavelength ( $0.6328\mu$  in the case of a helium-neon laser) is much less than the inner scale,  $l_0$ , it is hard to conceive that there would be a significant change in phase over a propagation pathlength of less than the inner scale,  $l_0$ . This approximation is referred to in the literature as the Rytov approximation.

In order to solve Equation (27) the following change of variable is made. Let

$$W = \Psi_1 e^{\Psi_0} = \Psi_1 E_0 \quad (28)$$

Therefore

$$\Psi_1 = \frac{W}{E_0} = W e^{-\Psi_0} \quad (29)$$



Now substitute Equation (29) into Equation (27) and expand terms to get

$$\nabla^2 W + k^2 W = -2k^2 n_1 E_o. \quad (30)$$

This differential equation has the solution

$$W(\bar{r}) = \int_{V'} 2k^2 n_1(\bar{r}') E_o(\bar{r}') G(\bar{r}-\bar{r}') dV', \quad (31)$$

where

$$G(\bar{r}-\bar{r}') = \frac{e^{jk(\bar{r}-\bar{r}')}}{4\pi(\bar{r}-\bar{r}')} \quad (32)$$

and is called Green's function for a point source emanating a spherical wave. [Ref. 10]

Finally, making the reverse substitutions, the solution for  $\Psi_1(\bar{r})$  is

$$\Psi_1(\bar{r}) = \frac{W}{E_o} = \frac{1}{E_o(\bar{r})} \int_V 2k^2 n_1(\bar{r}') E_o(\bar{r}') G(\bar{r}-\bar{r}') dV'. \quad (33)$$

Equation (33) with  $n_1(\bar{r})$  as a random, continuous function characterizes the scintillation in the propagating wave if  $n_1(\bar{r})$  is known analytically. However, it is not practical to measure the index of refraction at every point in the optical propagation path. Instead the approach will be to characterize  $n_1(\bar{r})$  in terms of the covariance function  $C_n(r)$ . In order to do so the statistics of  $\Psi_1(r)$  must be calculated starting from Equation (33) and the covariance of the refractive index given by Equation (7b).

In a rather long and involved process, Schmeltzer [Ref. 7] obtained an expression for the logarithmic amplitude covariance,  $C_{1A}^2(d)$  for a beam





propagating in a random medium. Fried [Ref. 8] reduced Schmeltzer's equation to

$$C_{1A}^2 = 0.124k^{7/6}Z^{11/6}C_N^2 \quad (34)$$

$$= \langle [\ln(A) - \ln\langle A \rangle]^2 \rangle \quad (35)$$

where  $Z$  is the optical pathlength and  $A$  is the amplitude of the wave functions  $|E(r)|$ . The logarithmic intensity variance is a much more tractable quantity to measure. Since intensity is the square of the amplitude.

$$C_{1I}^2 = \langle [\ln(A^2) - \ln\langle A^2 \rangle]^2 \rangle. \quad (36)$$

Expanding Equation (36) yields

$$C_{1I}^2 = \langle [2\ln(A) - 2\ln\langle A \rangle]^2 \rangle = 4C_{1A}^2 \quad (37)$$

Finally, substituting Equation (34) into Equation (37) and solving for  $C_N$  yields

$$C_N = 1.42k^{-7/12}Z^{11/2}C_{1I} \quad (38)$$

The laser scintillometer described elsewhere in this paper measures  $C_{1I}$  directly. As a result the scintillation power spectrum can be predicted since  $C_N$  is easily derived from  $C_{1I}$ .



### C. MODULATION THEORY

As is indicated by Equation (33), the effects of scintillation on the modulation content of a laser beam result in a rather complicated relationship. The presence of localized random intensity fluctuations in the beam which are, in the limit, capable of cutting the beam off suggest that scintillation noise is not a simple additive term in the expression. Hoversten, et. al. [Ref. 11] suggests that atmospheric channel effects can be treated as a multiplicative disturbance with log normal distribution. Hoversten described the channel output field,  $U_o(t, r_o)$  in the form

$$U_o(t, r_o) = ZS(t - \frac{L}{C})Z(t, r_o, L), \quad (39a)$$

or

$$U_o(t, r_o) = ZS(t - \frac{L}{C})\exp[\gamma(t, \bar{r}_o)]. \quad (39b)$$

where  $S(t)$  is the modulated intelligence in the beam,  $L/C$  accounts for the propagation delay, and the factor  $Zz(t, r_o, L)$  is a random process representing the turbulent effects ( $Z$  is a normalization constant so that  $|z|^2 = 1$ ).  $L$  is the propagation distance and  $\bar{r}_o$  is a two dimensional position vector in the output plane. Hence turbulence is represented by a multiplicative effect in this model.

The model to be used in this paper has been developed from an intuitive approach to the problem suggested by Thomas [Ref. 12]. It retains the multiplicative assumption of the Hoversten model but neglects the propagation delay and uses a different description of the atmospheric



noise. The signal independent background noise is assumed white with constant spectral density, say  $N_0$ , and will be neglected in this development.

Assuming the more general case of less than 100 percent intensity modulation of the beam, the amplitude modulation in the noise free case will be of the form

$$m(t) = k[A + B \cos(\omega_m t + \theta)], \quad (40)$$

where  $k$ ,  $A$ ,  $B$  are arbitrary constants and  $\theta$  is a uniformly distributed random variable with probability density function given by Equation (38).

$$\rho(\theta) = \begin{cases} \frac{1}{2\pi} & 0 < \theta < 2\pi \\ 0 & \text{elsewhere} \end{cases} \quad (41)$$

Since the model must hold in the near ideal case of short range and negligible scintillation, it must reduce to the ideal case of Equation (40) under these conditions. With the foregoing in mind, the output of the signal channel, i.e., the detector output signal model was assumed to be of the form

$$M[t, G(t)] = K[A + B \cos(\omega_m t + \theta)][G_1 + G(t)]. \quad (42)$$

$G(t)$  is a random process independent of  $\theta$ , which will describe atmospheric scintillation effects on the information content of the beam and  $G_1$  is the dc level of the noise. From the expected value definition of the autocorrelation function, i.e.,



$$R(\tau) = E[f(t)F(t+\tau)] \quad (43)$$

the autocorrelation of  $M[t, G(t)]$  can be calculated.

$$R_m[t, G(t)] = K^2 E_G \{ [G_1 + G(t)] [G_1 + G(t + \tau)] \} \quad (44)$$

$$X E_\theta \{ [A + B \cos(\omega t + \theta)] [A + B \cos(\omega_m [t + \tau] + \theta)] \}$$

Expanding Equation (44) and collecting the non-zero terms yields

$$R_m(\tau) = \frac{K^2 A^2 G^2}{2} + \frac{K^2 G_1^2 B^2}{2} \cos \omega_m \tau + K^2 A^2 G(\tau) \quad (45)$$

$$+ \frac{K^2 B^2}{2} G(\tau) \cos \omega_m \tau$$

under the following assumptions:

- (1)  $G(t)$  and  $\theta$  are independent
- (2)  $G(0) = 0$ , while still allowing  $G(0^+)$  to be non-zero.

Since the power spectral density of a function is just the Fourier transform of its autocorrelation function, transforming Equation (45) yields

$$\phi_m(\omega) = \frac{K^2 A^2 G^2}{2} + \frac{K^2 G_1^2 B^2}{2} [\delta(\omega - \omega_m) + \delta(\omega + \omega_m)] + \frac{K^2 A^2}{2} \phi_G(\omega) \quad (46)$$

$$+ \frac{K^2 G_1^2 B^2}{2} [\phi_G(\omega - \omega_m) + \phi_G(\omega + \omega_m)].$$





$\phi_m(\omega)$  is the power spectral density of the signal plus scintillation noise received at the detector and therefore is the result of interest.

The foregoing is a brief summary of some of the major steps in the development of the governing equations for the application of intelligence to a propagating spherical wave in a random medium. For the complete derivation of the propagation equations, the original papers by Tatarski [Ref. 6], Smeltzer [Ref. 7], and Fried [Refs. 8 and 9] should be consulted.

The modulation theory presented in this paper was developed using an intuitive approach to the problem. The major assumption underlying the theoretical development is that in the time domain, scintillation phenomenon has a multiplicative effect on intelligence impressed on a laser beam. Such an assumption of multiplication in the time domain is necessary to explain frequency translation effects observed in the preliminary experiments.



#### IV. EXPERIMENTAL APPARATUS

##### A. EXPERIMENTAL SETUP

The experimental setup for ship-to-shore testing in the maritime environment shown in Figure 2 is the final evolution of this phase of the experiment, before integration with the full atmospheric laser project being conducted by the Physics and Meteorology Departments of the Naval Postgraduate School.

The heart of the atmospheric data collection system used by the Physics Department team is the laser scintillometer described in detail by Haagensen, [Ref. 2] The main components of the system are illustrated in Figure 4. Basically the scintillometer consists of two optical detectors. One of the detectors is focused directly on the laser beam and the other is focused out of the beam to detect background radiation from other than the scintillation source ( $0.6328\mu$  laser). The outputs of the two detectors are combined in a differential amplifier to subtract the background radiation and return only the laser output. The signal, now containing only the instantaneous intensity variations of the laser, is logarithmically amplified to get a signal proportional to the log of the intensity. The signal is next fed to a wideband power amplifier to boost it to the levels required by an instrumentation recorder.

As originally conceived, the setup consisted of a JODON model HN-15, 15 mw, HeNe laser operating at  $0.633\mu$ , which was to be used as the scintillation source for the joint project. For the communications experiments the laser was to be modulated by an acoustoptic modulator, designed by the author, which consisted of a distilled water cell exited by a gold plated quartz transducer. The modulator, under ideal conditions



was capable of modulating the beam to a depth of approximately 100 percent with a maximum of 25 percent of the laser power appearing in the modulated beam. The higher power laser, as opposed to the 1.0 mw laser described elsewhere in this paper, was used in anticipation of up to 5 km ship to shore modulation experiments from the research vessel. While the 15 mw laser with accoustooptic modulation was able to deliver sufficient power for the joint project (2-5 mw) under optimum conditions, the required Bragg deflection angle offset and laser power fluctuations as a result of shipboard vibration caused system alignment difficulties which became a threat to the entire project. The optical receiver used in the embryonic stages of this project consisted of the MERET FDA425 hybrid detector/amplifier mounted at the focal point of a 2" diameter, f5, uncorrected lens.

The vibration problem which arose because the laser mirrors were mounted separately from the laser tube, was the reason for demounting the laser scintillation source from the shipboard stable platform and moving it to an instrument trailer setup on the shoreline. At the same time, the scintillometer was removed from the instrument trailer and redesigned to fit on the stable platform aboard the research ship. It was felt that the project would benefit considerably from having the full power of the 15 mw scintillation source available which meant that the accoustooptic modulator would have to be replaced by a device which allowed much more efficient use of the laser power. It was decided that an electrooptic modulator, manufactured by Coherent Associates, would be used as soon as available. Another problem that arose in conjunction with reversing the relative positions of the scintillation source and the scintillation receiver was the location of the optical receiver within the limited weight and size constraints of the stable platform. The



original f5 receiver which was 12" long, weighed 2 lbs. and required an effective mounting area of  $9\text{in}^2$  was redesigned to an f2.4 system which was 4" long and required an effective mounting area of  $2\text{in}^2$ . The design of the seagoing experiment was complete with the redesign of the optical receiver to fit on the stable platform. The electrooptic modulator was not available in time however to complete the experiment.

In order to collect data for this paper, the seagoing experiment was simulated in a roof top experiment on June 5, 1974. The experiment diagramed in Figure 2 was setup on a 300 m folded range of two 150 m legs. A 1 mw METROLOGIC model M/L 699 modulatable laser was mounted on a tripod and focused on a 10 x 16 in flat mirror. The optical receiver was mounted in the totally returned beam on another tripod and was used to collect the data represented by Graphs 5 to 13.

#### B. THE LASER AND MODULATOR SYSTEM

The laser used for this experiment was a ML-669 modulatable laser manufactured by METROLOGIC Corporation. The laser modulation circuitry consists of a preamplifier, a modulator driver and a current amplifier. The modulator circuitry acts to control the d.c. discharge current in the laser tube and thereby control the intensity of the beam. The laser is specified to have a modulation depth of 15 percent, but the 1.0 v peak to peak maximum signal to the input produced a modulation depth of slightly over 20 percent. The laser was rated at 0.8 mw optical power. Actual power out was measured to be 1.0 mw. The specified bandwidth of the laser modulator is 300 Hz to 500 KHz. The measured bandwidth was somewhat wider, 200 Hz to 580 KHz, with usable components below 10 Hz. Specified beam divergence was 0.8 mrad while the actual measured divergence was less than 0.7 mrads.





### C. THE OPTICAL RECEIVER SYSTEM

The optical receiver used in the experiment consisted of a MERET FDA425 hybrid detector/amplifier combination mounted at the focal point of an one inch diameter f 2.4 uncorrected lens. The specified optical characteristics of the detector are a spectral range of  $0.36\mu$  to  $1.15\mu$  with a peak responsitivity greater than 15 millivolts/microwatt at  $0.905\mu$ . The responsivity at  $0.633\mu$  was  $\sim 9$  mv/ $\mu$ w. The electrical bandwidth was dc to 10 MHz and the rise time was less than 40 nsec. The optical receiver system driving a 40 db Hewlett-Packard Model 450DA wideband amplifier was capable of detecting signals of less than 40 nw with a signal to noise ratio of greater than 5:1. During the experiment, over a range of 300 meters, the signal power at the detector was approximately  $20\mu$ w.

### D. SIGNAL PROCESSING EQUIPMENT

The data for the experiment, collected on magnetic tape, was processed on a Federal Scientific Corporation model UA-500 Ubiquitous Spectrum Analyzer. The machine stores and speeds up the input signal enabling rapid spectrum analysis by step hetrodyning techniques. The signal in storage is Fourier analyzed and presented at output terminals in both the frequency and time domain. The analysis process in the machine involves passing the signal through as many as several hundred contiguous synthetic filters. As a result the resolution elements (effective 650 line resolution) possess a stability and uniformity unattainable by other techniques. One of the more useful features of the machine is its spectrum averaging capability which allows the adding and ensemble averaging of as many as 1024 statistically independent spectra in real time. These spectra can also be averaged in a peak or exponential mode.



## V. DATA ANALYSIS

The data represented by Figures 5 through 13 was recorded between 2200 and 2300 on June 5, 1974. The scintillation spectrum (without modulation) shown in Figure 1 was recorded for three minutes at the beginning of the hour and is assumed valid for the entire period. The scintillation spectrum as can be clearly seen from Figure 5 does not exhibit the  $-11/3$  slope as predicted by Equation (2). There are number of possible reasons for the departure from the predicted value. The most likely explanation is that a larger building exhaust fan was blowing at right angle to the beam. It is theorized that the fan was adding localized turbulent energy to the atmosphere at some point, probably 36 Hz, in the inertial subrange. The low frequency end of the spectrum in Figure 1 as well as in Figures 2,4,6 and 8 is degraded by the low frequency rolloff of the Ampex SP-300 data recorder. The SP-300 has a lower frequency 3 db point at 10 Hz and a rolloff slope 720 db/octave.

The ambient scintillation indicated in Figure 5 extends to 400 Hz. Since amplitude modulation requires a double bandwidth, the data which resulted from the experiment will be discussed using twice the scintillation spectrum or 800 Hz as a standard.

Figure 6 is the spectrum of the detected signal when the laser was amplitude modulated 20 percent with a 50 Hertz sinusoid. The signal to noise ratio is very near unity but a close comparison of Figure 6 with Figure 5 shows that the upper frequency limit is almost 600 Hz. There is also an upward shift in the lower frequency components of the curve. It seems obvious that a frequency translation mechanism is in effect, however the model, Equation (43), predicts an upwards shift of only 50 Hz and not the almost 200 Hz observed.



Figure 7 is the constant amplitude oscilloscope synchronization signal from the signal generator.

Figure 8 is the spectrum of the detector output with the beam modulated 20 percent with a 100 Hertz sinusoid. While Figure 8 shows a somewhat better signal to noise ratio, as compared to Figure 6, there is no appreciable frequency translation. Since the frequency translation from 50 to 100 Hertz represents only an 8 percent change in the 600 Hertz spectrum, the change is difficult to distinguish on the logarithmic scale of a CRT X-Y plotter and is negligible when displayed by a mechanical plotter.

Figure 9 is the synchronization channel spectrum of the signal generator used to modulate the laser.

Figure 10 is the spectrum of the detector output with the beam modulated 20 percent with a 1000 Hertz sinusoid. Compared with Figures 6 and 8 it is a rather dramatic example of frequency translation on scintillation noise. Comparing the spectrum represented by Figure 10 with the model of Equation (46) it is seen that there is a large component at the dc level which is predicted by the squared, frequency independent, term,  $\frac{K^2 A^2 G^2}{2}$ , of Equation (46). The scintillation noise is clustered about the low end of the spectrum as would be expected and is predicted by the  $\frac{K^2 A^2}{2} \phi_G(\omega)$  term of the equation. The noise translation to a spectrum above and below the carrier (represented by the delta function) is predicted by the  $\frac{K^2 G^2 B^2}{2} [\phi_G(\omega - \omega_0) + \phi_G(\omega + \omega_0)]$  term. While Equation (46) is adequate to predict the general shape of the spectrum, it does have shortcomings in predicting the observed bandwidths of the scintillation spectrum. As is shown in Graph 1 the scintillation bandwidth is 400 Hertz, however, the bandwidth of the lower frequency components in Figure 10 which presumably is the scintillation, has unaccountably increased to





600 Hz. Also the model of Equation (43) predicts that the translated noise from the low end of the spectrum would have a bandwidth of 1200 Hz about the carrier. Such is not the case in Figure 10. The observed bandwidth is only 600 Hz total.

Figure 11 is the signal generator synchronization signal.

Figure 12 is the resultant spectrum which results when the carrier is far out in the spectrum from the scintillation noise. This particular graph shows the best "fit" to the model as compared with the previous three spectra. The translated noise spectrum is about the right bandwidth compared with the spectrum of the low end of the frequency scale and this appears reasonably close (although still somewhat high) to the same width as indicated by Figure 5. The low level signal shown at approximately 2 KHz is spurious as far as all records of the experiment indicate and no firm explanation of its presence can be offered since it appears on the monitor spectrum of Figure 13 at 840 Hz. Since the signal is there though, it is obvious that it was translated upward in frequency along with the noise and there is indications of the scintillation translation in the upward translated spurious spike.

Figure 13 is the constant amplitude signal generator oscilloscope synchronization signal.





## VI. CONCLUSIONS

One objective of this research was to design and test a system to experimentally measure the effects of atmospheric scintillation on atmosphere AM laser communications. With the completed design of the seagoing system described earlier, this particular objective has been substantially accomplished. As with any other system which is used in relatively new applications, earlier designs evolve into later ones as the need for a particular capability is established or as a shortcoming is discovered. One particular shortcoming that the author predicts will have to be faced early on is the large and unwieldy Ampex SP-300 data recorder which will be hard to manage and control on the rolling and pitching deck of the research vessel.

The second objective of this research was to develop a predictive model to describe the effects of atmospheric scintillation on atmospheric AM laser communications. The model given by Equation (46) comes very close to making that prediction. The presence of every portion of the received signal is predicted by the model and it can serve now as a usable expression for preliminary design work. As the Data Analysis section of this paper shows, the shortcoming of Equation (46) bandwidth of the translated noise spectrum. Such a shortcoming raises as many questions as it answers. The basic development of Equations (40) through (46) and the assumptions made to simplify Equation (46) are open to question. Consideration of the effects of the bandwidth limitations of the instrumentation is suggested for further study. Another possibility is that the ambient atmospheric scintillation changed enough over the



period of the experiment to significantly change the level that was assumed constant due to the building exhaust fan blowing across the transmission path.

The questions raised by this paper are there for follow on researchers on the project and the author wishes them much success.



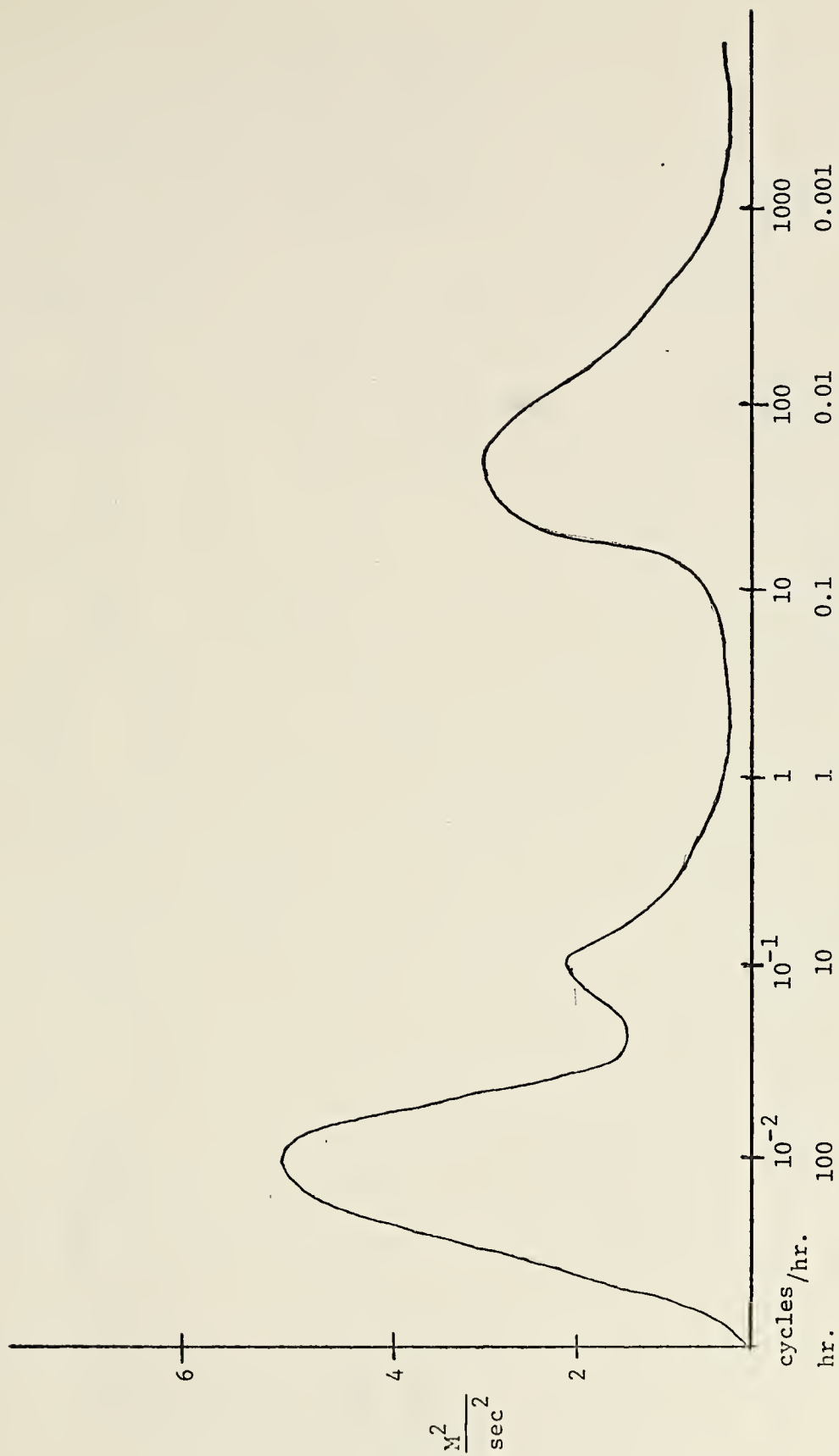


Figure 1. Spectrum of Horizontal Wind velocity (Van der Hoven, 1957)



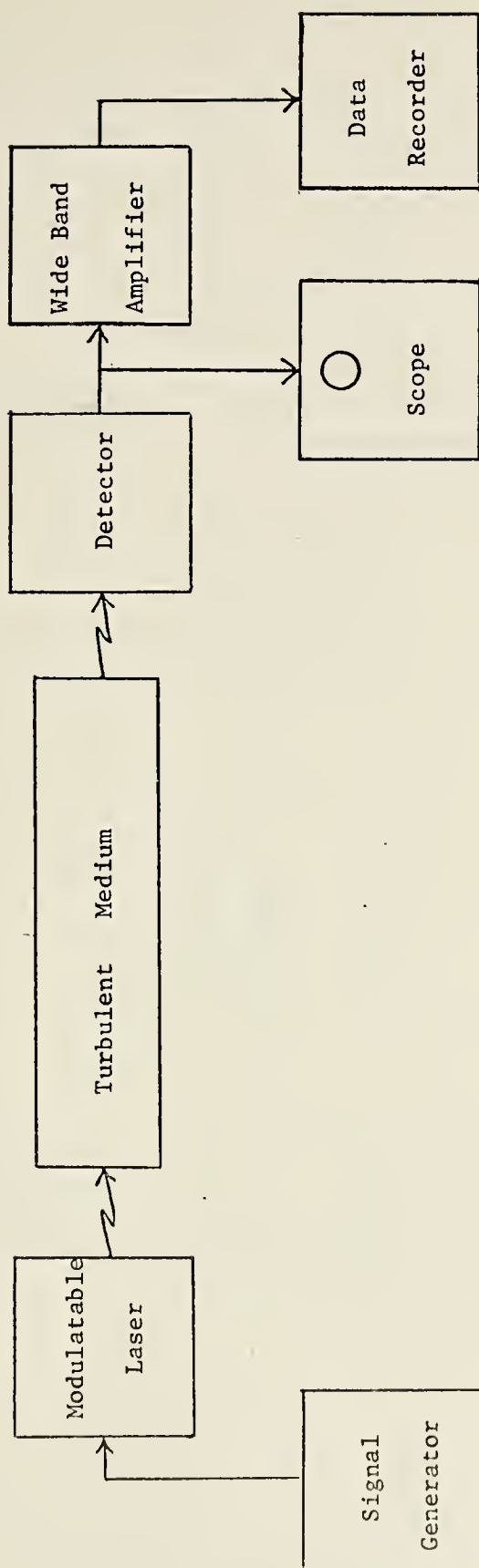


Figure 2. Experimental Setup





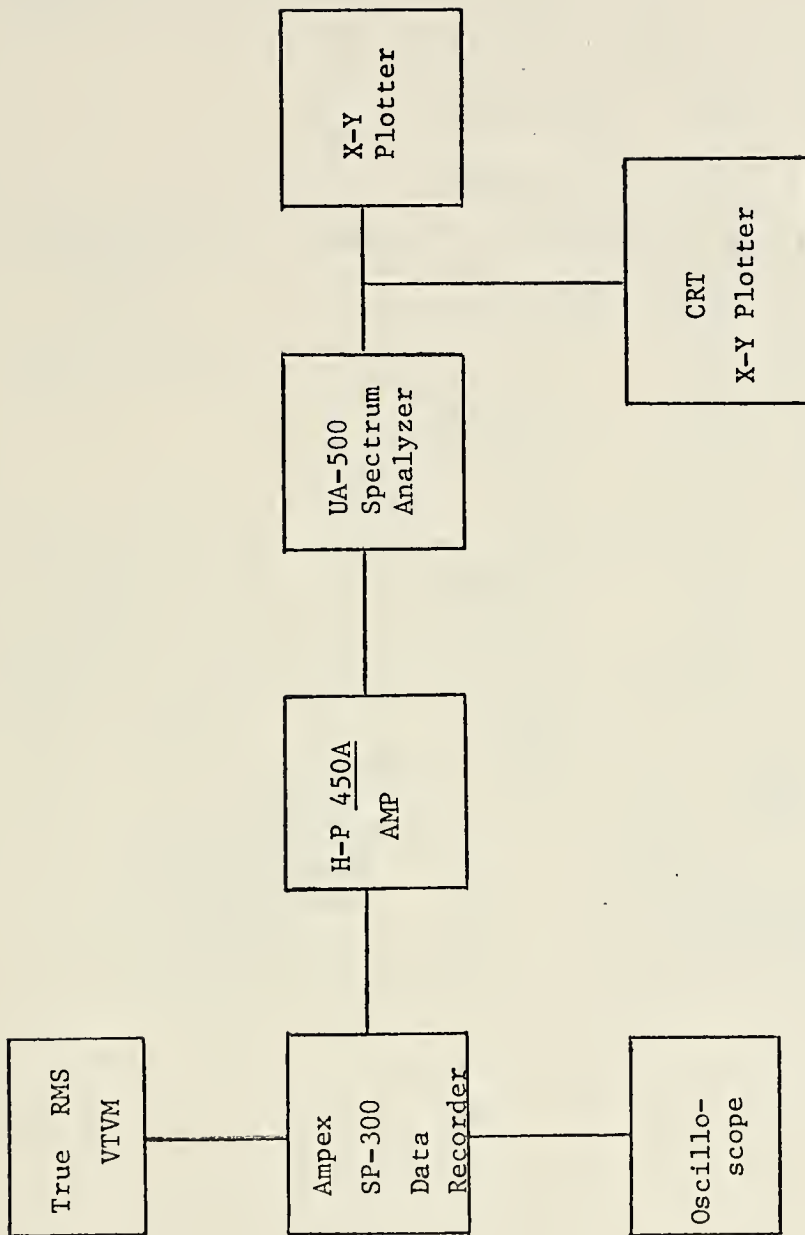


Figure 3. Data Analysis System



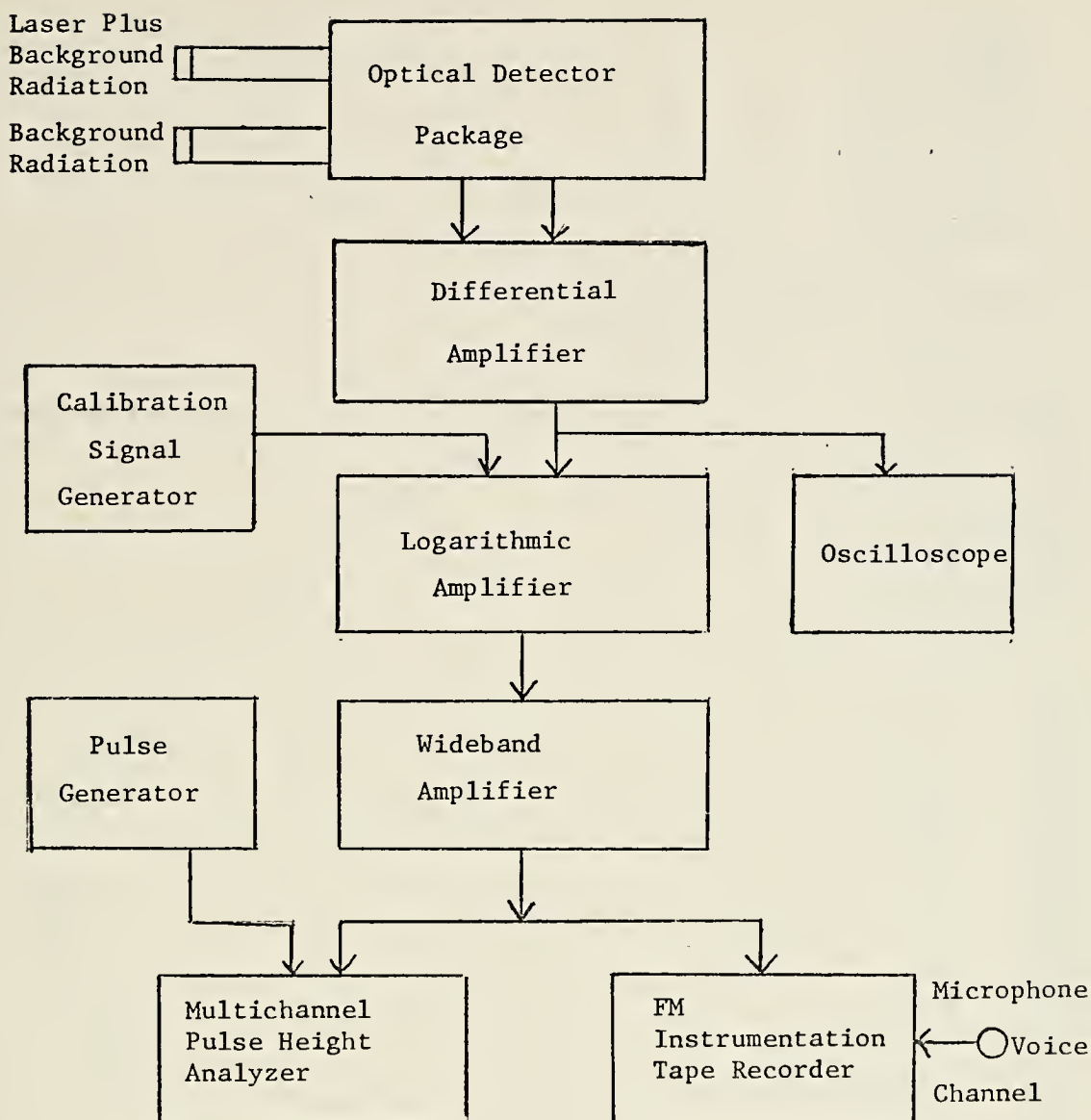


Figure 4  
The Laser Scintillometer System



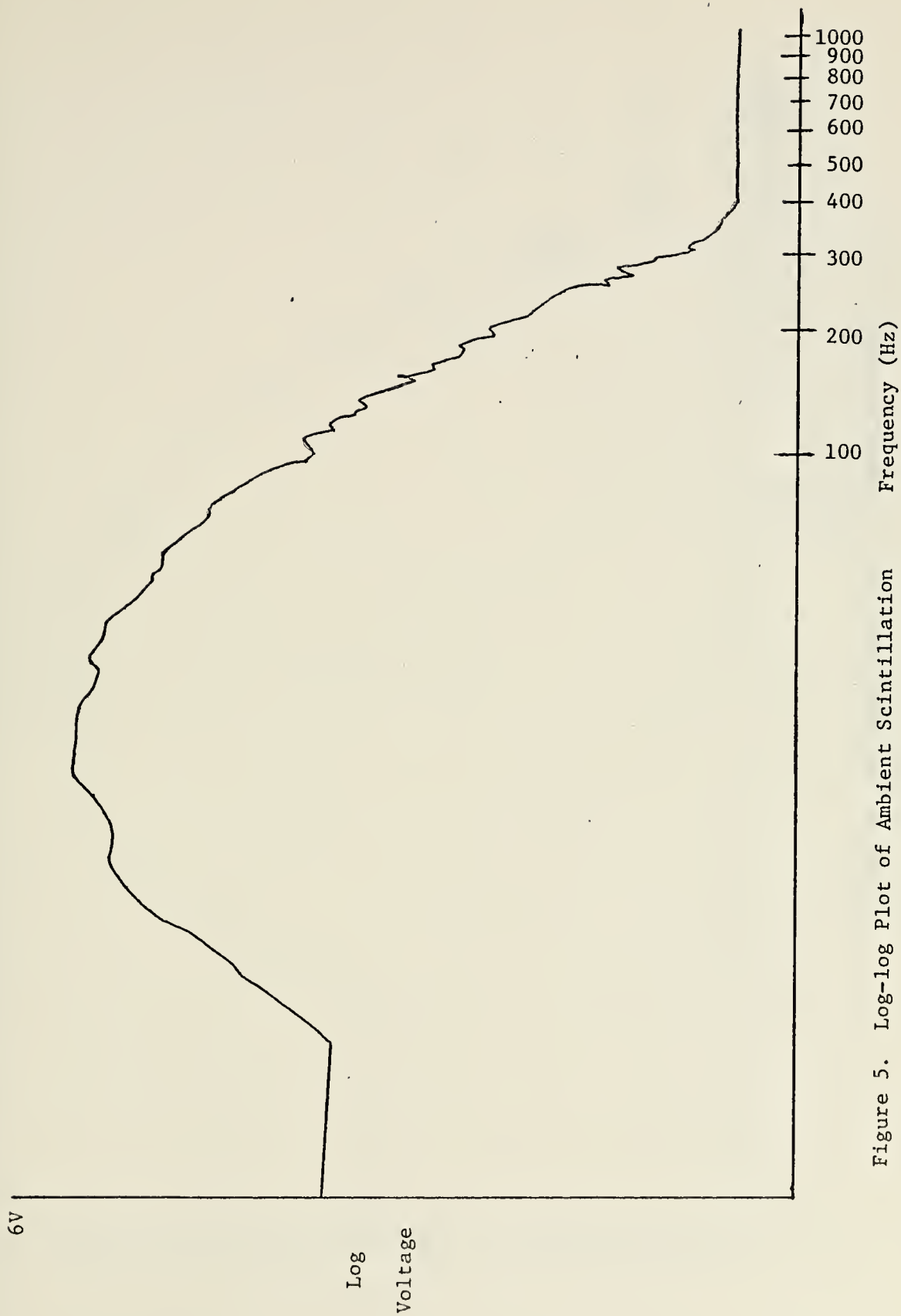


Figure 5. Log-log Plot of Ambient Scintillation



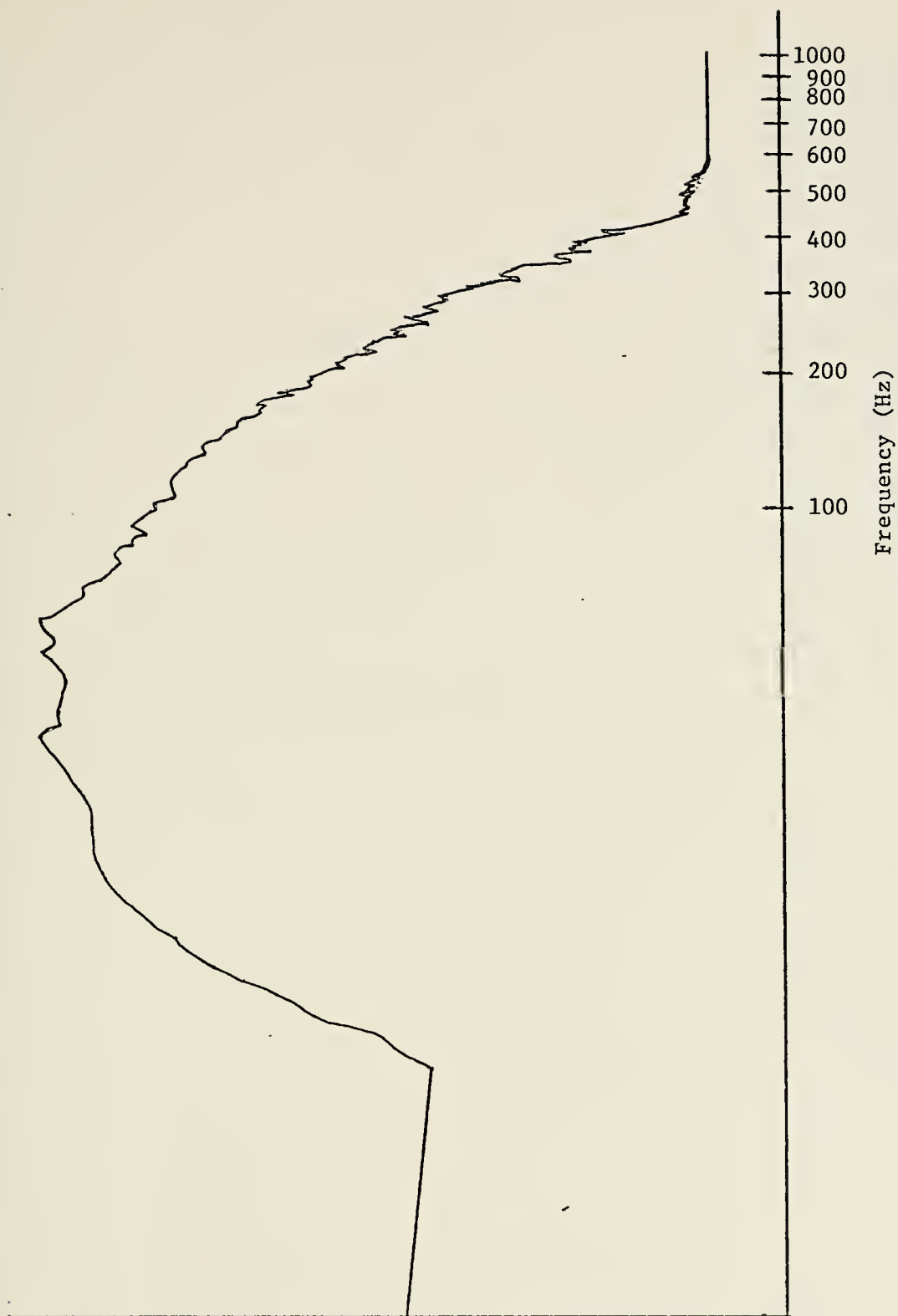


Figure 6. Log-log Plot of 50 Hz Signal in Scintillation





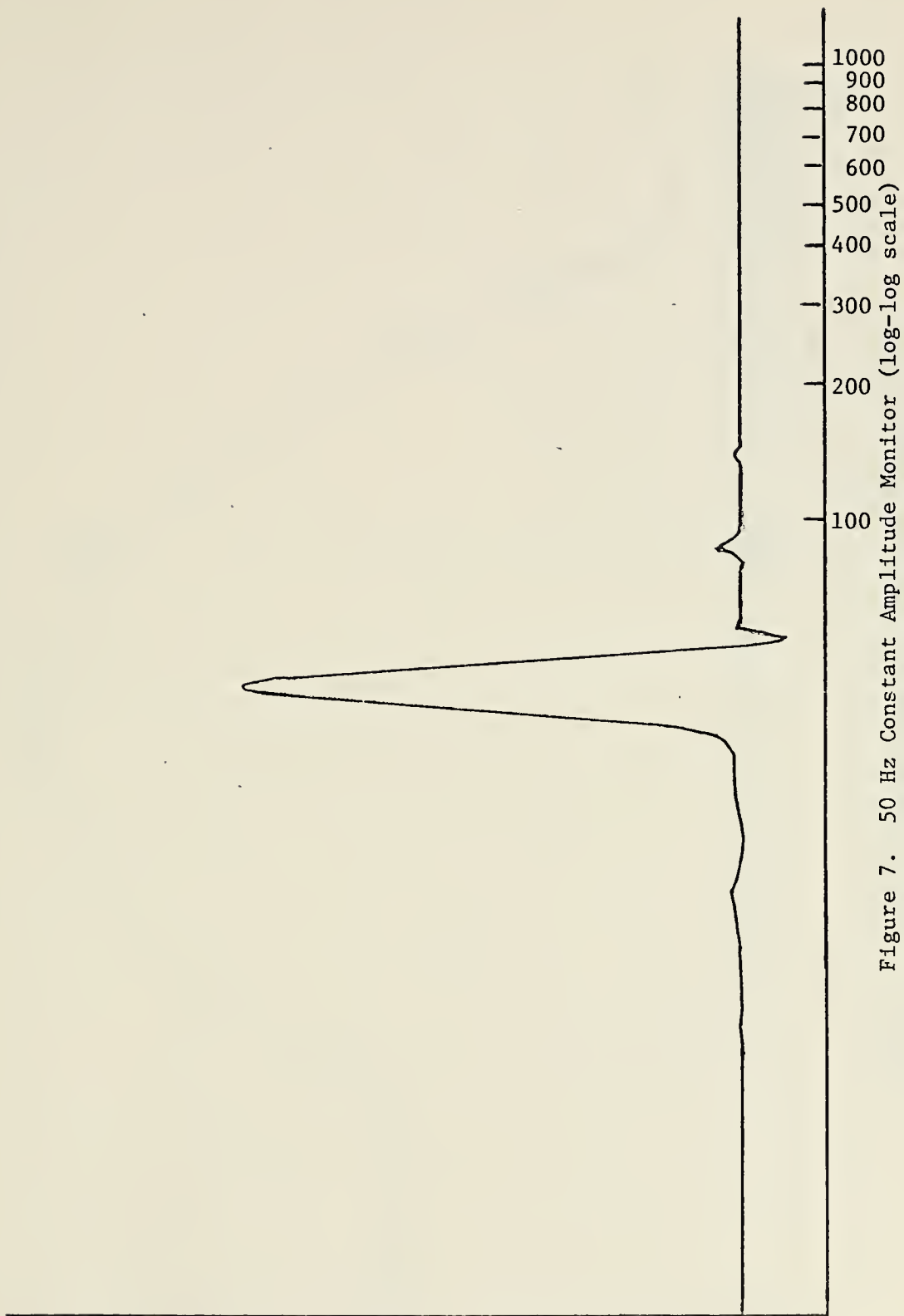


Figure 7.



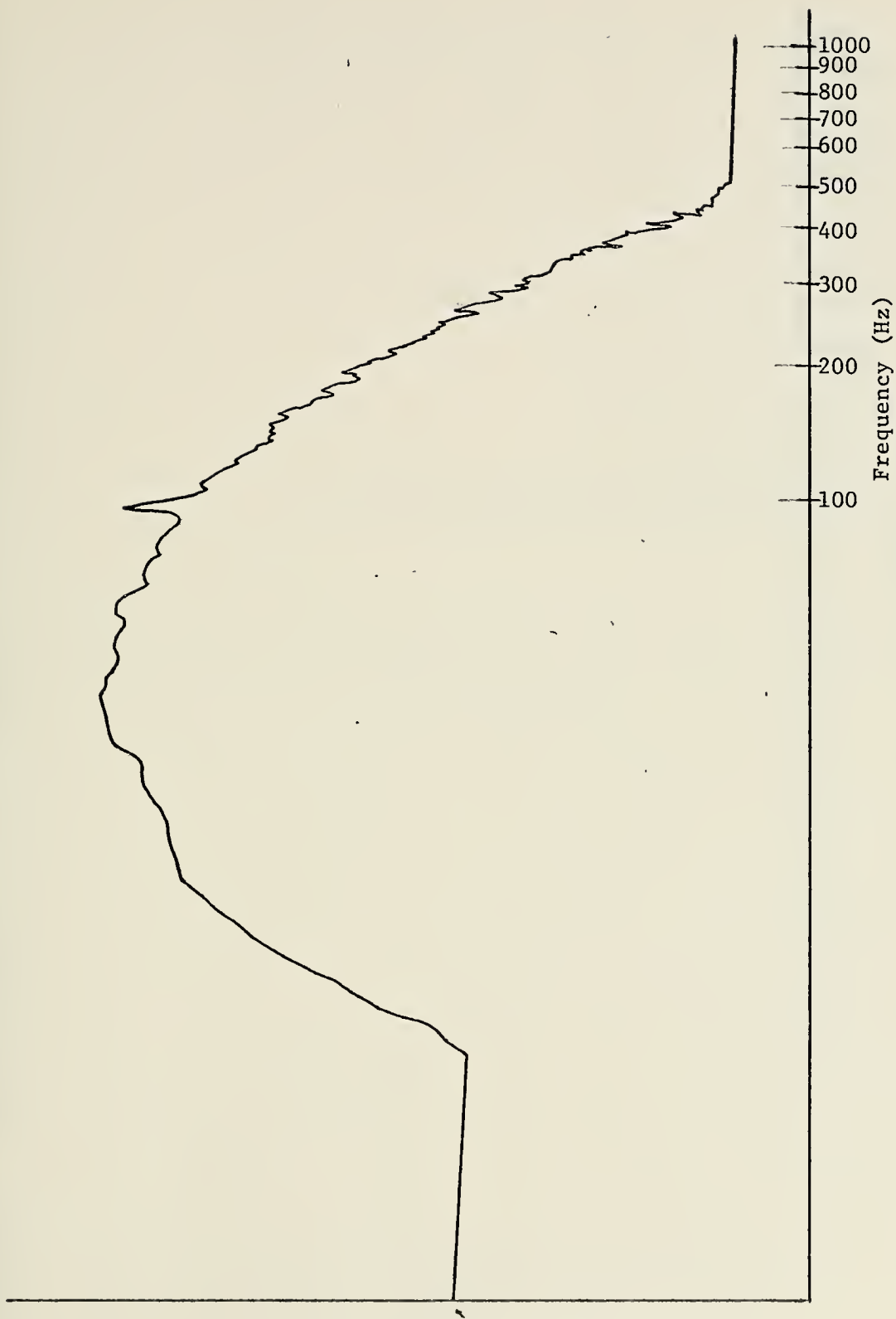


Figure 8. Log-log Plot of 100 Hz Signal in Scintillation



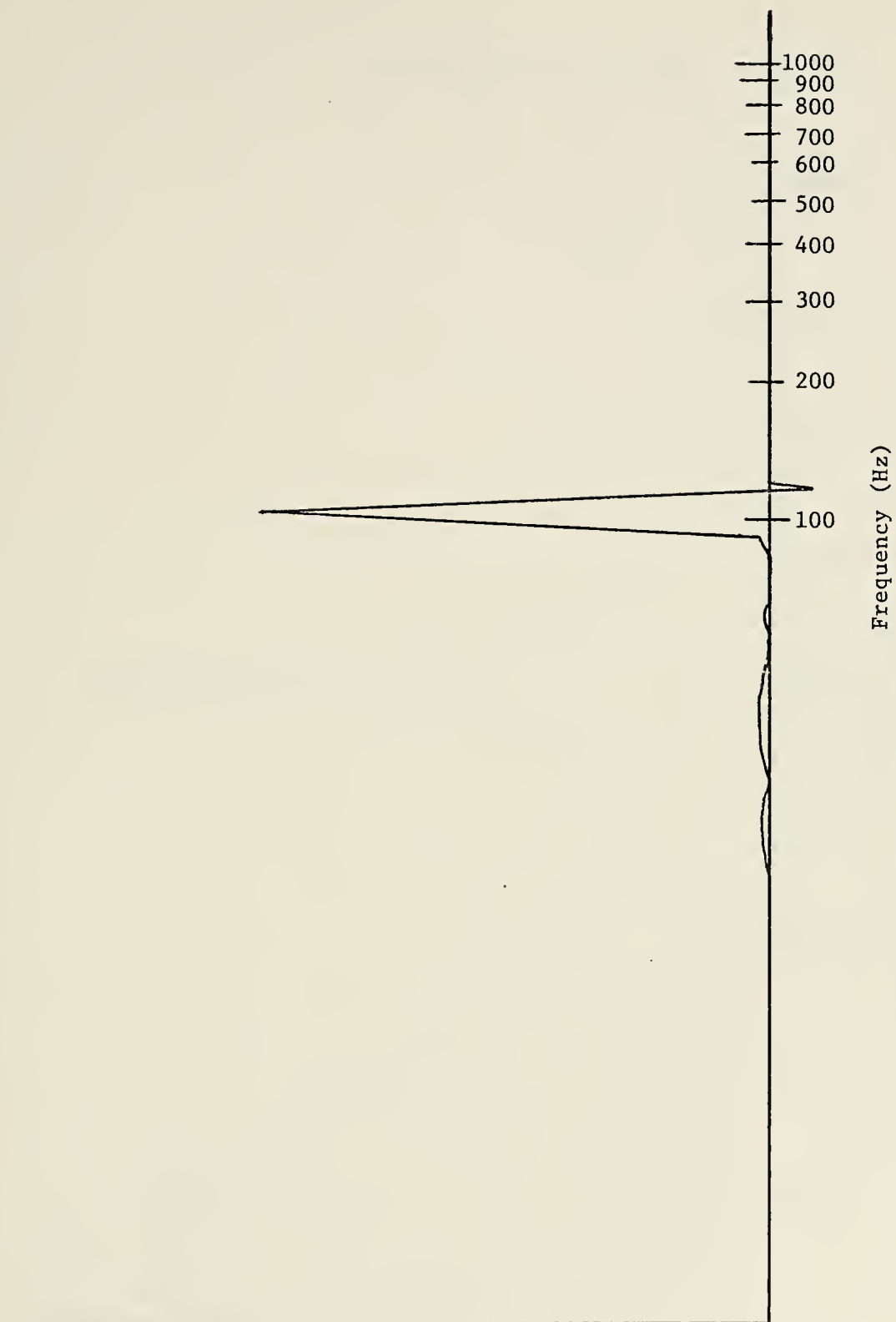


Figure 9. 100 Hz Constant Amplitude Monitor Plot



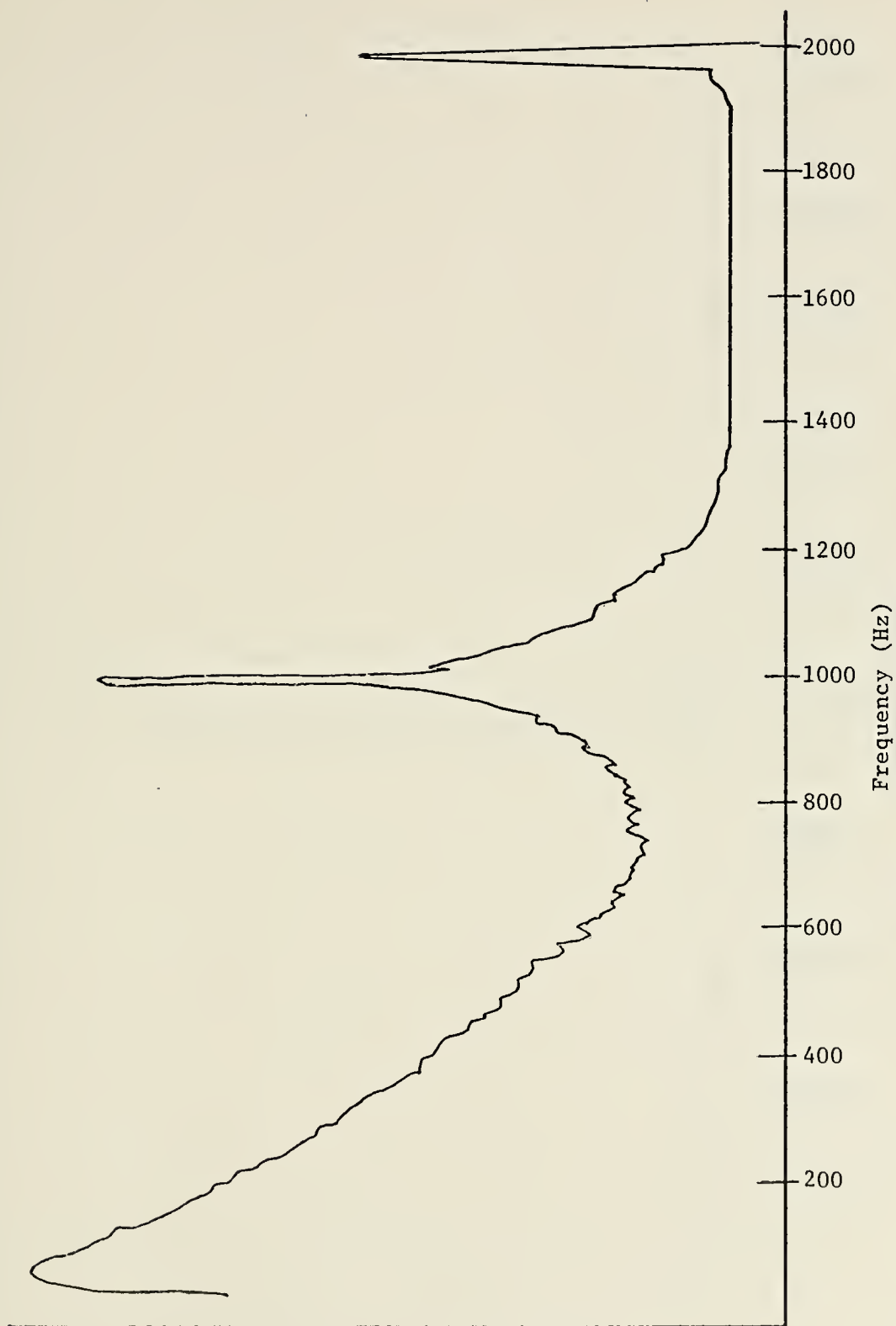


Figure 10. Semi-log Plot of 1 KHz Signal in Scintillation





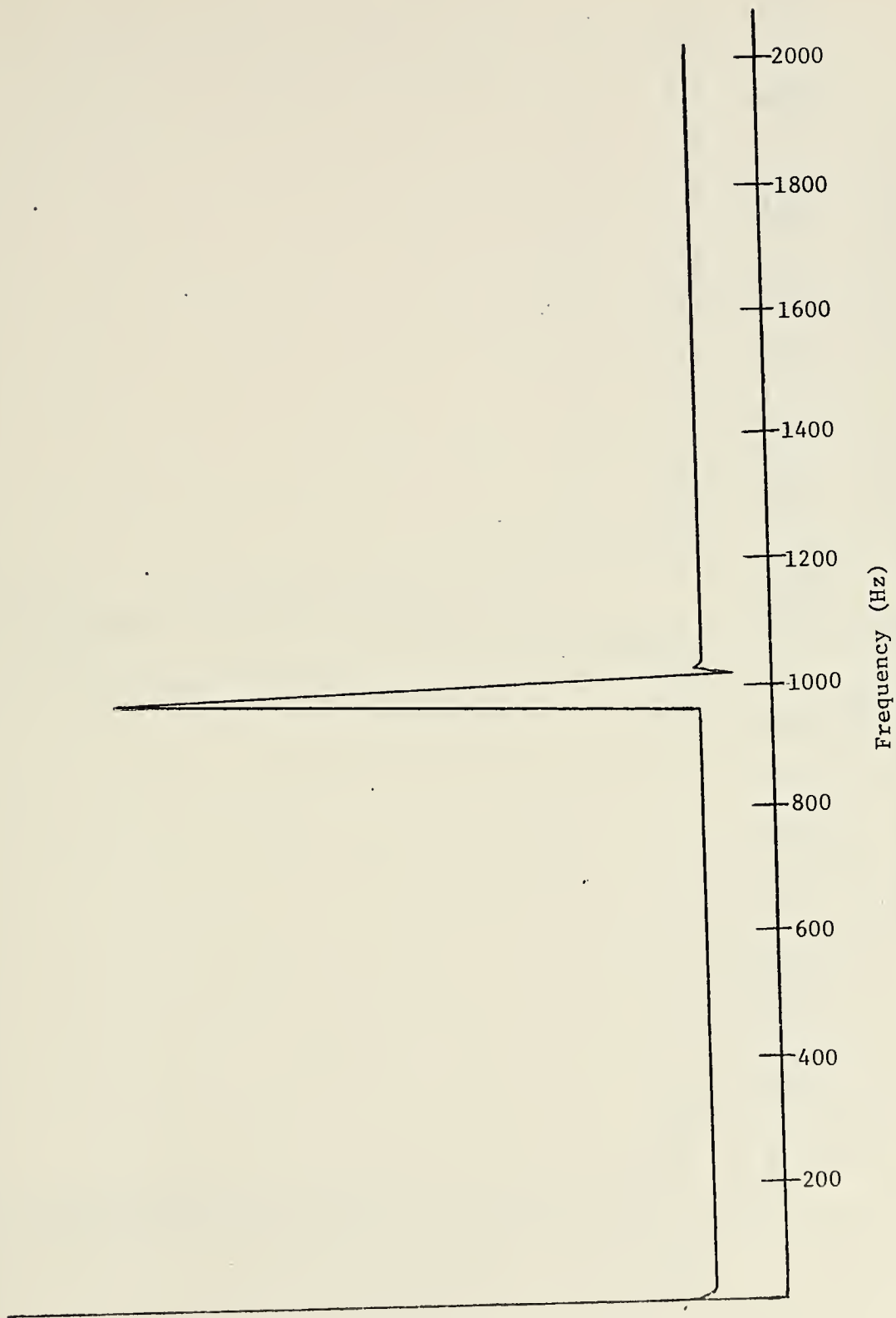


Figure 11. 1000 Hz Constant Amplitude Monitor Plot.



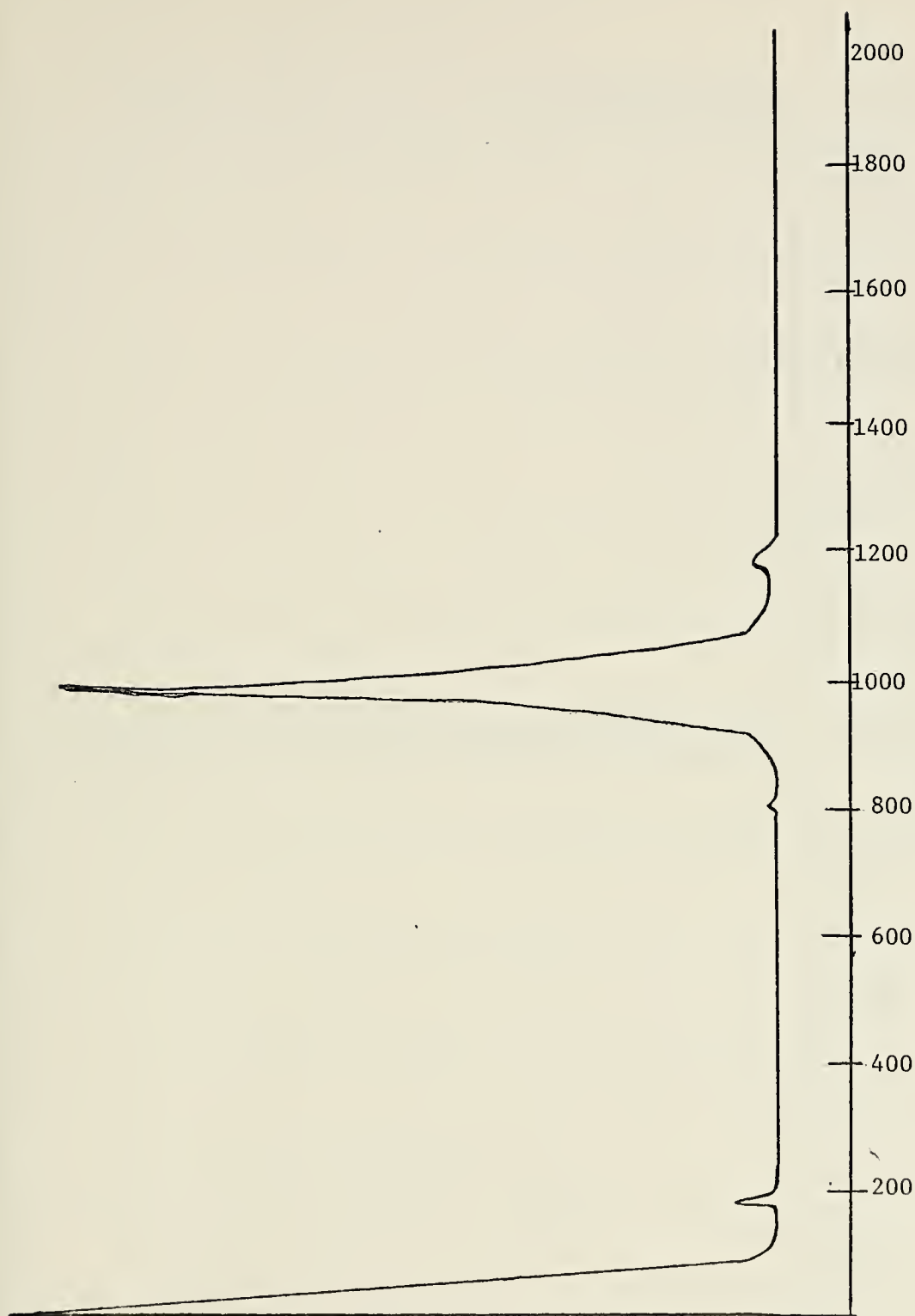


Figure 12. Semi-log Plot of 10 KHz Signal in Scintillation.



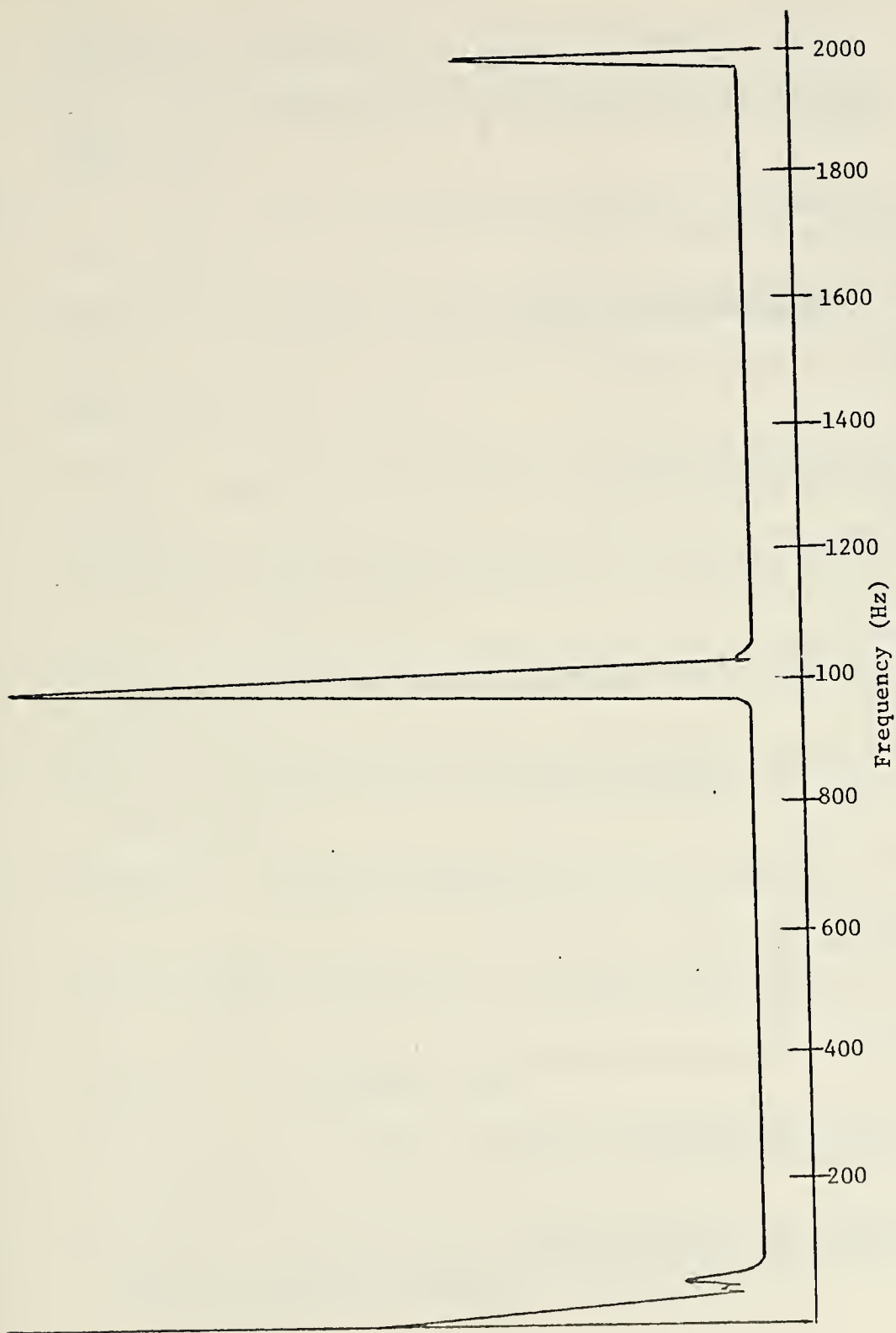


Figure 13. 10 KHz Constant Amplitude Monitor Plot.



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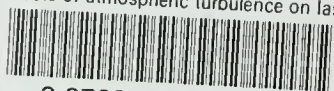
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